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# Sumário

١.	Fisolofia	Social	E	Teoria	DO	Direito	

1	. Logica e ontologia das normas Juliano Souza de Albuquerque Maranhão	7					
2	. <i>Jus Cogens</i> como aporia: o crepúsculo do direito interna- cional clássico SAMUEL RODRIGUES BARBOSA	39					
3	. O ceticismo dos realistas norte-americanos: a indetermi- nação no direito CARLA OSMO	93					
4	. Hermenêutica jurídica e a questão da textura aberta José Maria Arruda de Andrade	138					
II. Lógica e Filosofia da Ciência							
1	. Relative charity FABIEN SCHANG	159					
2	. Logics of deontic inconsistency Marcelo E. Coniglio	173					
3	. Biconditional drive to paradox Jean-Yves Béziau	196					
4	. Condicionalidade e a lógica de implicação normativa de Von Wright JULIANO SOUZA DE ALBUQUERQUE MARANHÃO	202					
III.	Ontologia e Filosofia dos Valores						
1	. Dualidade e a estrutura do mundo Alexandre Costa-Leite	233					
2	. Entre as veredas da cultura e da civilização José Luiz Borges Horta e Marcelo Maciel Ramos	248					

### 6 REVISTA BRASILEIRA DE FILOSOFIA 2009 – RBF 233

## IV. Traduções

1.	A grande tradição Hannah Arendt	281							
2.	Razão prática e traços de caráter: um comentário à teoria sentimentalista de Maccormick sobre a percepção moral CLAUDIO MICHELON	296							
V. Resenhas de Obras, Temas e Autores									
1.	John Rawls e o utilitarismo Gabriel Bertin de Almeida	313							
2.	Law and the Media: a theoretical and methodological criti- que. Gies, Lieve. 2008. Law and the media – The future of an uneasy relationship. Routledge-Cavendish/Glass Hou- se, 166 p. MARIA FRANCISCA CARNEIRO	328							
3.	La governanza de internet Antonio A. Martino	334							
Membros da Diretoria									
Membros Efetivos									
Normas de Publicação para Autores									

### 3 Biconditional drive to paradox

Jean-Yves Béziau

Professor da UFC.

#### ÁREA DO DIREITO: Filosofia

**RESUMO:** O artigo<sup>1</sup> traz uma discussão sobre o caráter paradoxal do bicondicional clássico, de acordo como o qual, dadas mais de duas proposições, a disjunção de todos os pares de bicondicionais é uma tautologia.

**PALAVRAS-CHAVE:** Bicondicional – Paradoxo – Fórmula de Dugundji's – Lógica modal – Lógica poli-valente. **ABSTRACT:** This is a discussion about the paradoxical character of classical biconditional according to which, giving more than two propositions, the disjunction of all pairs of biconditionals is a tautology.

**KEYWORDS:** Biconditional – Paradox – Dugundji's formula – Modal logic – Many-valued logic.

The scene takes place in the Tea Room of the Philosophy Department of Hollywood University.

Bill: How is it going?

Tom: Pretty good, and you?

Bill: Not so bad, but I'm a bit perturbed by a paradox I came across last night while studying Dugundji's theorem to improve my knowledge about modal logic.

Tom: One more paradox! Let us see if I find a solution to this one.

1. Work supported by a grant DCR (Funcap/CNPq) and within the Log-Comp research project (CNPq). Thanks to all members of Log Comp and also to Rodrigo Bacelar, Alexandre Costa-Leite, David Makinson and Pedro Santos. Bill: Tom, as far as I know, you like cars, you already had plenty of them.

Tom: That's right, but I don't see any paradox there.

Bill: Sure, that's perfectly ok. Now do you think that it is the same to drive a Cadillac, a Rolls Royce or a Ferrari?

Tom: Surely not, unless you are a pretty woman.

Bill: But that is what classical logic says.

Tom: I am puzzled; I love very much classical logic. But I love cars too and if classical logic says something wrong about cars, I may change classical logic for a paraconsistent turbo polar logic or another crazy logic. But can you explain me how classical logic leads to this strange affirmation?

Bill: A theorem of classical logic is that given three propositions p1, p2, p3, the proposition  $p1 \leftrightarrow p2 \lor p2 \leftrightarrow p3 \lor p1 \leftrightarrow p3$  is a tautology.

Tom: I have never noticed that, maybe because I have never focused on bi-conditional, generally I deal with conjunction, disjunction, negation, conditional and sometimes some funny things like Sheffer stroke.

Bill: So according to this tautology it means that given three cars, at least it is the same to drive two of them. And, since classical logic is formal, this is true for any kind of things or situations, for example giving three shapes such as sphere, triangle and square, it is the same for the earth to have one of these two shapes, for example to be triangular and spherical.

Tom: It is really amazing, so in mathematics, it means that given three propositions such as the axiom of choice, the continuum hypothesis and Fermat's theorem, at least two of them are equivalent.

Bill: That's right. Let us take an even simpler example among the three propositions: 1+2=0, 1+2=3 and 1+2=78, at least two of them are equivalent.

Tom: But in this case it seems not so absurd since two of them are false. We can say that two false propositions are equivalent, because 1+2=0 has the same level of absurdity as 1+2=78, even if the first looks like a cheaper absurdity.

Bill: Yes, that's the line of thought that can maybe save classical logic. Given three propositions at least two are true or two are false,

so we can say that two are equivalent in the sense that they have the same truth-value. But when we are saying that "Tom is driving a Rolls Royce" is the same as "Tom is driving a Ferrari", because the two propositions are false, it seems we are losing something.

Tom: What are we losing? I hope not my driving license!

Bill: Losing meaning. Classical logic is not a logic of meaning, maybe it is a meaningless logic.

Tom: Then maybe we shall shift to modal logic, relevant logic or another kind of intensional logic.

Bill: But note that the tautology  $p1 \leftrightarrow p2 \lor p2 \leftrightarrow p3 \lor p1 \leftrightarrow p3$  is also valid in modal logic. What is not valid in a modal logic like S5 is the disjunction of necessitation of bi-conditionals, for example  $(p1\leftrightarrow p2)\lor(p2\leftrightarrow p3)\lor(p1\leftrightarrow p3)$ , called the 3-Dugundji formula.

Tom: It seems then that modal logic is not a meaningful logic either. But why people call S5 an intensional logic and think that it is fine not to have the validity of the 3-Dugundji formula, but having the validity of the nude 3-Dujundi formula – if I may call it like this –  $p1 \leftrightarrow p2 \lor p2 \leftrightarrow p3 \lor p1 \leftrightarrow p3$ ?

Bill: That's really a mystery, or maybe just a lack of consciousness of what is going on. Nevertheless one may think that the dressed Dugundji formula is worse, if we consider the endomorphic nature of necessity.

Tom: Necessity may be anthropomorphic, but in which possible world would it be endomorphic?

Bill: This just means that the operator of necessity embeds in the object language the meta-logical notion of logical truth. So the dressed 3-Dugundji formula means that among any two propositions, two are logically equivalent, by contrast to the nude formula just saying that they are bi-conditionally equivalent, and even less since it is modulo classical disjunction, and in classical logic we don't have like in intuitionistic logic:  $p \lor q$  is a tautology if, and only if, p is tautology or q is a tautology.

Tom: Do you mean that two propositions p and q are logically equivalent in the sense that whenever the proposition p is true, the proposition q is true and vice-versa?

Bill: Yes, whatever the circumstances: it is true for you that you are driving a Ferrari if and only if it is true that you are driving a Cadillac, but in the object language of modal logic also modulo classical disjunction.

Tom: What kind of circumstances? The important thing is not that it is true or not, but if it is the same thing, modulo classical disjunction or not.

Bill: Endomorphic modal logic, like classical logic, has no meaning. That makes sense, since the idea is just to enrich the object language with the meta-language which is also meaningless. Note, however, that the validity of the dressed Dugundji formula would be worse in the sense that classical logic does not say that any two propositions among three are logically equivalent.

Tom: So how to go out of this trap? Maybe by using many-valued logic? It looks like bivalence is driving us to this paradox, reducing three to two.

Bill: But multiplication of values does not drive us very far. If you take a finite number of values, the paradox will repeat: if you have 3 values, putting an additional car in your garage, you will find two among the four, say a Toyota and a Ferrari such that it is the same that you are driving a Toyota and you are driving a Ferrari.

Tom: Thanks for putting an additional car in my garage; I would have preferred a Lamborghini. But what is the third value, besides truth and falsity, you are adding? Suppose that it is possibility. Then what is the problem if I say that it is the same, that it is possible that I drive a Toyota, and that it is possible that I drive a Ferrari?

Bill: The question is that driving a Toyota and driving a Ferrari is the same since they have the same truth-value and having the same truth-value would naturally lead to truth, or better to distinguished truth, so that the nude 4-Dugundji formula will be true, or better distinguishingly truth.

Tom: I see, I remember a Polish friend of mine who pointed out that in many-valued logic, bivalency is still there through the distinction between distinguished and non-distinguished values. That is why in fact I was convinced one more time to stick to good old classical logic. But let's drive to the infinite. Imagine that we have an infinite numbers of truth-values, then you can add as many cars as you want in my garage, driving two of them can always be different.

Bill: Not necessarily: imagine we have a denumerable numbers of truth-values and that I put in your garage a non-denumerable number of cars, then it will be possible to find two cars such that it is the same to drive them and we can drive endlessly in this way in Cantor's paradise.

Tom: Cantor's paradise is very nice and I wish I was living there with a non-denumerable number of cars even if it is the same to drive two of them. But in reality it is not like this, I have a finite number of cars and in fact there is no nude  $\omega$ -Dugundji formula, which can hold in Cantor's paradise since such formula does not exist.

Bill: It is true that in standard logic there is no nude  $\omega$ -Dugundji formula and that we can construct a logic with infinite values such that no nude *n*-Dugundji formula is valid for any *n*. Are you satisfied with that?

Tom: Not completely, but it is better than classical logic where driving a Ferrari is the same as driving a Toyota.

Bill: Maybe driving to the infinite is just a way of escaping the problem. What we need is a meaningful way of constructing a logic where the nude *n*-Dugundji formula is not a tautology for any natural number *n*.

Tom: Did you find such a construction?

Bill: Not yet, I am exploring. But anyway this biconditional drive to paradox convinced me that classical logic is wrong.

Tom: I remember an Ozzie logician who enjoyed very much to say all the time, especially before lunch, that classical logic is wrong. But how do you explain that if classical logic is so wrong, so many people are still studying it?

Bill: Syllogistic was studied during many years and some people like Kant thought it was eternal. It seems to me that classical logic is nowadays in the same position and many modern Kants claim that first-order logic is eternal. The success of classical logic is based on repetition and the fact that it is never confronted to reality and never effectively used except by some people dealing with angel sexuality as it was the case with syllogistic. Tom: Angel sexuality is nice, but if classical logic is not effective to drive my car I prefer a shift of logic. Imagine I have a car crash based on the use of classical logic, it would be terrible. Do you think it is possible?

Bill: Maybe or maybe not, because a simple logic, even false, can be enough. In the same way that to drive your Ferrari you can rely on Newtonian physics and you don't need Einstein's theory of relativity.

Tom: Alright, so I will keep thinking that I can safely drive my Ferrari with classical logic, and also my other cars. By the way, I didn't tell you that I recently bought a Porsche. What about driving you to lunch with this new car, you will feel the difference.

Bill: ok, vive la différance! And that's nice because I had just a flat tyre with my bicycle this morning.